Yaroslavl State University
Preprint YARU-HE-95/04

hep-ph/9511366

# Unitary Limit on $K_L^0 \to \mu^+\mu^-$ and the Top Quark Mass

Gvozdev A.A., Mikheev N.V. and Vassilevskaya L.A.
*Yaroslavl State University, Yaroslavl 150000, Russia*

**Abstract**

A brief overview of the recent measurements of the branching ratio of the rare $K_L^0 \to \mu^+\mu^-$ decay in the context of their agreement with the Standard Model (SM) is given. It is shown that KEK results well correlate with the SM and B-physics, whereas the BNL results are in conflict with the SM with the heavy top quark.

For quite a long time the rare electroweak decays $K_L^0 \to \mu^+\mu^-$ and $K_L^0 \to \gamma\gamma$ have been the subject of intensive investigations. An observation of the heavy top quark with the mass $\sim 190\,GeV$ [1, 2] and a further precision of the measurement of $V_{ub}$ and $V_{cb}$ CKM matrix elements improved by ARGUS and CLEO [3] make a curious situation about the rare electroweak decay $K_L^0 \to \mu^+\mu^-$. It is known that the connection between the absorptive part of the $K_L^0 \to \mu^+\mu^-$ decay width and $K_L^0 \to \gamma\gamma$ decay width gives us the down limit of the probability of the $K_L^0 \to \mu^+\mu^-$ decay:

$$Br_{abs}(K_L^0 \to \mu^+\mu^-) \simeq 1.2 \cdot 10^{-5}\, Br(K_L^0 \to \gamma\gamma) = (6.8 \pm 0.3) \cdot 10^{-9}. \quad (1)$$

Here we used the experimental value of $Br(K_L^0 \to \gamma\gamma) = (5.73 \pm 0.27) \cdot 10^{-4}$ [4]. This minimal value allowed by theory is known as the unitarity limit.

In our papers [5] we have investigated the processes $K_L^0 \to \mu^+\mu^-$ and $K_L^0 \to \gamma\gamma$ within the quark model approach to obtain the estimation of the top quark mass. In this approach the $K_L^0 \to \mu^+\mu^-$ amplitude is the sum of one-loop ($1L$) and two-loop ($2L$) contributions. The first one (through $W$ and $Z$) due to short distances $\sim 1/m_W$ where the top quark contribution dominates. As for the $2L$ contribution with two photons in the intermediate state, both intermediate ($\sim 1/m_c$) and rather long ($\geq 1/m_K$) distances are essential in it.

We pointed out the principal importance of the correct relative sign of the $1L$ and $2L$ contributions in the amplitude. Let us note that in the terms of the bare quarks the total decay amplitude contains these contributions with opposite signs [6]. However we emphasize that it is necessary to account the QCD corrections to the effective four-quark vertex to obtain a realistic result for $2L$ contribution. To that end we used the renormgroup method by Vainstein, Zakharov and Shifman [7] for the mass scale $\mu$ down to the typical hadronic scale $\mu_0 \simeq 2\Lambda$ ($\alpha_{st}(\mu_0) = 1$). We have developed also a fenomenological method of the estimation of the QCD corrections in the region $\mu_0 \leq \mu \leq m_K$ [5]. To test the reliability of our method we estimated the ratio $\Gamma(K_L^0 \to e^+e^-\gamma)/\Gamma(K_L^0 \to \mu^+\mu^-\gamma)$ [8] and showed that our result was in agreement with one obtained within the phenomenological pole model [9]. Certainly we do not pretend to obtain an integral accuracy better than 30% in the description of the contributions of relatively long distances ($r \leq 1/\mu_0$). However, the sign between the $1L$ and the real part of the $2L$ contributions is fixed sufficiently reliable by this way.

Our main result is that the real part of the $2L$ contribution changes the sign if the QCD corrections are taken into account. The change of the sign is connected with the behaviour of the integral over the $u$-quark loop scale. This integral involves multiplicatively the QCD formfactor of an effective four-quark $(V-A)$ vertex which becomes sufficiently large (more than unit in modulus) and negative number in the region $2\Lambda \leq \mu \leq m_K$ [7].

The expression for the total $K_L^0 \to \mu^+\mu^-$ amplitude obtained by this way has the form [5]:

$$
\begin{aligned}
\mathcal{M}(K_L^0 \to \mu^+\mu^-) &\simeq -10^{-3}\mathcal{N}\{+[(5.6\pm 2.0) - i(44.7\pm 0.9)] \\
&\quad +2 + 10^3\,\frac{F(m_t^2/m_W^2)}{\sin^2\theta_W}\,\frac{\Re(V_{td}^* V_{ts})}{\Re(V_{cd}^* V_{cs})}\}, \\
F(x) &= \frac{x}{4}\left[\frac{4-x}{1-x} + \frac{3x\ln x}{(1-x)^2}\right],
\end{aligned}
\tag{2}
$$

where $\mathcal{N} = (\alpha/4\pi)\,G_F\,F_K\,m_\mu \sin 2\theta_C(\bar{\mu}\gamma_5\mu)$, $F_K$ is the formfactor of the $K$-meson, $m_\mu$ is the muon mass, $\theta_C$ and $\theta_W$ are the Cabibbo and Weinberg angles, respectively; $F(x)$ is the well-known function [10], The first term in the curly braces describes the $2L$ contribution. We pretend only on the calculation of the real part of the $2L$ contribution, and take the imagine part from the unitarity relation (1). The second and third terms of the amplitude (2) describe the $c$- and $t$-quark contributions, respectively.

It should be noted that our expression (2) for the total decay amplitude is in contradiction with the calculation of Ko [11] in which short and long distance contributions have opposite signs. The method developed by Ko [11] has, in our opinion, disadvantage. Namely, in these papers the dependence of the meson vertex formfactors (for example, $\pi VV$) on the meson loop scale was neglected.

To obtain the restriction on the $K_L^0 \to \mu^+\mu^-$ decay width, we used the resent experimental data on the top quark mass of the CDF Collab. [1]

$$m_t = 176 \pm 8(\text{stat.}) \pm 10(\text{sys.})\, GeV/c^2$$

and on the parameters of CKM matrix in the Wolfenstain representation [3]

$$
\begin{aligned}
&\lambda = \sin\theta_C \simeq 0.22, \qquad A = 0.80 \pm 0.12, \\
&\sqrt{\rho^2+\eta^2} = 0.36 \pm 0.14 \qquad \text{which gives} \qquad (1-\rho) \geq 0.64 \pm 0.14.
\end{aligned}
$$

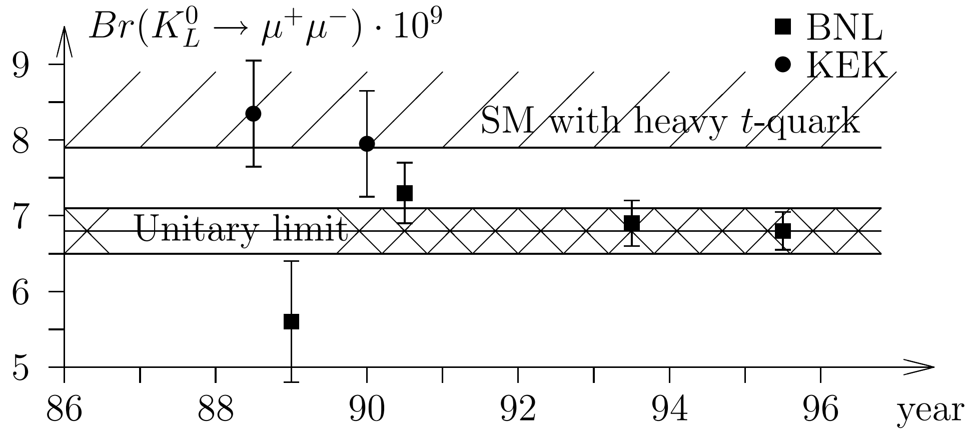


Figure 1.

On Fig.1 we present the experimental data of the measurement of $Br(K_L^0 \to \mu^+\mu^-)$ of BNL E791 Collab. [12] and KEK E137 Collab. [13]. As illustrated in Fig.1, the KEK results well correlate with the SM and $B$-physics, whereas the BNL results are in conflict with the SM with the heavy $t$-quark. If the BNL result is verified by new series of more precise measurements it may be a signal of a new physics beyond the SM. For example, the real part of the total amplitude (2) can contain a contribution of the relatively light leptoquark [14] which can cancel sufficiently the contribution of the heavy top quark.

The research described in this publication was made possible in part by Grant No. RO3300 from the International Science Foundation and Russian Government. The work of Mikheev N.V. was supported by a Grant N d104 by International Soros Science Education Program. The work of Vassilevskaya L.A. was supported by a fellowship of INTAS Grant 93-2492 and is carried out within the research program of International Center for Fundamental Physics in Moscow.

## References


[1] CDF Collab., F.Abe et al., Preprint FERMILAB-PUB-95/022-E (1995);

[2] D0 Collab., S.Abachi et al., Preprint FERMILAB-PUB-95/028-E (1995).

[3] A.Ali and D. London, Z.Phys.C. 65 (1995) 431.

[4] Particle Data Group, L. Montanet et al., Phys. Rev. D50 (1994) 1173.

[5] A.A.Gvozdev, N.V.Mikheev and L.A.Vassilevskaya, Phys. Lett. B 274 (1992) 205; Yad. Fiz. (Sov. J. Nucl. Phys.) V.53 (1991) 1682;

[6] M.B.Voloshin and E.P.Shabalin, Sov. Phys. JETP Lett. V. 23 (1976) 107.

[7] A.I.Vanstein, V.I.Zakharov and M.A.Shifman, Zh. Eksp. Teor. Fiz. 72 (1977) 1275.

[8] A.A.Gvozdev, N.V.Mikheev and L.A.Vassilevskaya, Phys.Lett. B292 (1992) 176.

[9] L.Bergstrom et al., Phys.Lett. B249 (1990) 141.

[10] E.Ma and A.Pramudita, Phys. Rev. D22 (1980) 214;
T.Inami and C.S.Lim, Progr. Theor. Phys. 66 (1981) 297.

[11] P.Ko, Phys.Rev. D44 (1991) 139; D45 (1992) 174.

[12] A.P.Heinson et al., Phys.Rev. D51 (1995) 985.

[13] KEK E137 Collab., T.Akagi et al., KEK preprint 94-151 (1994).

[14] A.V.Kuznetsov and N.V.Mikheev, Phys.Lett. B329 (1994) 295.